Centrifuge Modeling of Levee Breaches: Is it possible?

J.L. Wibowo and B.A. Robbins
U.S. Army Engineer Research and Development Center, Geotechnical and Structures Laboratory, Vicksburg, Mississippi, USA

S.C. Wilhelms and D.A. Leavell
Bowhead Science and Technology, Vicksburg, MS, USA

ABSTRACT: The U.S. Army Engineer Research and Development Center has previously conducted centrifuge modeling of homogeneous levee embankments to investigate the impact of dynamic loading on levee stability. In some of these centrifuge experiments, an embankment breach was initiated such that the breach formation process could be observed. Remarkably, the scaled, centrifuge models of the embankment breaching process exhibited characteristics (head-cut erosion and side slope collapse) identical to those of prototype scale breach models conducted by the United States Department of Agriculture. While the objective of these experiments was not to study breach process, the observations from the tests prompted an in depth review of the feasibility of centrifuge modeling of levee breaching processes. The results of this review are presented, including a review of pertinent scaling laws as well as an overview of past studies. It was determined that centrifuge modeling may be a viable means of investigating levee breaches, and further model studies should be conducted to evaluate the scaling laws that may apply.

1 INTRODUCTION

The US Army Engineer Research and Development Center current research includes the study of the physical process of levee erosion caused by overtopping flow. Conventional experimental work, i.e., physical modeling based on Froude Number, on levee erosion can demonstrate erosion phenomena, but requires a large model to adequately represent actual erosion rates and patterns. This nearly full scale surface erosion model would be very expensive and require significant efforts, especially to evaluate a variety of soils, construction techniques, and flow conditions. It would be especially time consuming to establish saturated conditions in levees constructed from silty-clays. The smaller scale centrifuge models would be ideal in terms of cost and time if they obey the scaling laws. After centrifuge modeling became popular in geotechnical engineering, Goodings (1982) started to study the scaling laws of surface erosion processes. Goodings' follow-up studies were presented in 1984 and 1985. Since then, many researchers have studied the effects of scaling on this important subject.

2 OBJECTIVES

The objective of the effort described herein is to assess the potential and feasibility of centrifuge modeling of surface erosion of breached levees. The scope of this effort includes summarizing the physical forces and processes that dominate erosion, particularly, erosion during a levee breach. This includes a clear explanation of the distortion effects of conventional small-scale Froude modeling of erosion. Past uses of centrifuge modeling for investigating soil erosion or other surface flow phenomena, were reviewed. The potential effects of modeling under high-gravitational acceleration were investigated and evaluated relative to improving the capability for modeling erosion. The feasibility of conducting such physical modeling in the ERDC centrifuge was also assessed.

3 CONCEPTUAL DESCRIPTION OF EROSION PROCESSES

For most of the sediment transport modeling conducted in the past, the chief interest was in where scour and deposition occurred in a riverine or estuarine environment. It is likely that in most of these investigations, sediment was being transported from an upstream source to a downstream deposit without regard to the specific processes that eroded banks or soil surfaces to produce the sediment. In the investigation of levee surface erosion and subsequent breaching, the forces and processes would be much more localized and the understanding of these processes would be of much greater importance to the modeling of surface erosion phenomenon.

Consider the situation where a steady uniform overtopping flow initiates surface erosion on a levee. It is likely, even though the flow is steady and uniform, that small rills will form in the levee surface. Once a rill forms, additional erosion will widen and deepen the rill accelerating the water flow and rate of erosion. With rills deepening into small channels, erosion will occur at the head of the channel extending the channel to the “river” side of the levee, where a levee breach would then be initiated. With even a small channel completely through the levee, the rate of erosion will likely further accelerate deepening and widening the channel with increasing discharge.

The previous sections describe, in very general terms, the erosion progression that can ultimately cause a levee breach. However, this description does not include the finer details of the forces and process contributing to the soil erosion and movement. With uniform flow over a flat soil surface, the fluid exerts a tractive force on the soil particles due to the velocity gradient near the surface, which tends to move particles in the direction of the flow. The mean flow also tends to cause lift forces on particles that protrude up into the flow, which can, with increasing velocity, lift soil particles from the surface for transport downstream. These forces tend to be described by mean flow characteristics, such as average velocity, depth, and unit discharge.

The processes contributing to the erosion of a breach through a levee are all of those discussed in the previous sections, as well as others not included in this brief discussion, such as seepage flow through a levee. It would seem that physically modeling of all these processes would be necessary to reproduce the rate of erosion in a breach as well as the pattern of erosion. It is with this objective in mind that we examine the potential for high-g modeling.

The U.S. Army Engineering Research and Development Center has previously conducted centrifuge modeling of homogeneous levee embankments to investigate the impact of dynamic loading on levee stability. In some of these centrifuge experiments, an embankment breach was initiated such that the breach formation process could be observed. Remarkably, Figure 1 shows the scaled, centrifuge models of the embankment breaching process exhibited characteristics (headcut erosion and side slope collapse) identical to those of prototype scale breach models conducted by the United States Department of Agriculture (Hunt et al., 2005). While the objective of these experiments was not to study breach process, the observations from the tests prompted an in depth review of the feasibility of centrifuge modeling of levee breaching processes.

Figure 1. The headcut phenomena were occurred during dynamic load test on levee using centrifuge

4 DEVELOPMENT OF SIMILITUDE FOR CENTRIFUGE MODELING

Although there are many publications related to physical modeling of sediment transport in rivers and streams, based on our literature review, it appears that Goodings (1982, 1984, 1985) was one of the first to examine the modeling relationships to couple sediment movement due to subsurface flow (seepage) and surface erosion in high-gravity centrifuge modeling, in which she wrote three papers. In her 1982 article, she presents the development of the parametric relationships for scaling model and full-scale values of velocity, discharge, and time. For subsurface flow, a conflict arises between two different relationships for scaling time, when using the same fluid and soils in the model and full-scale prototype. She points out that others addressed this problem by altering the fluid from water to one of higher viscosity or reducing the effective particle size of the soil, thereby reducing permeability. For granular soils, the latter seems to hold greater potential for physical modeling. However, limits are likely regarding the changes to particle size, since there should be no significant effects on soil density, friction, and cohesion.

Goodings goes on to examine the parametric relationships for modeling surface flows, using the Reynolds Number and Froude Number, but imposes a discharge relationship from subsurface flow; since she initially assumes that the surface flow comes from seepage. As expected, she notes that the Reynolds Number of the model will be lower than the Reynolds Number of the full-scale prototype by a factor of the model scale; but like most hydraulic modeling, if the Reynolds Number of model flows is in the same regime as the prototype, i.e., laminar, transitional, or turbulent, then similitude will likely be preserved.

To model surface erosion, Goodings gave an "entrainment" function as the basis for scale development. This function is the ratio of shear stress exerted on the soil surface to the resistance of the average particle to erosion. Using this function for the scaling relationships, similitude is preserved only if the particle size in the model is reduced compared to the full-scale prototype. However, the reduction for surface erosion $\left(\frac{D_{50m}}{D_{50p}} = 1/N\right)$ is significantly greater than for seepage flow $\left(\frac{D_{10m}}{D_{10p}} = 1/\sqrt{N}\right)$, where N is the increase in gravity. Thus, Goodings concludes that seepage flow and soil transport can be simulated in centrifuge modeling by reducing effective particle size, but simultaneously modeling surface erosion "..will be difficult…"

In her 1984 article, Goodings expands her investigation of high-gravity erosion modeling, specifically examining the scaling laws for seepage, surface flow erosion, initiation of erosion, and the rate of sediment transport in a one-g model and then comparing them to similitude laws for high-g modeling. She reviewed the scaling laws for seepage flow in a conventional one-g and centrifuge model, noting the conflicts and solutions discussed earlier.

Of more interest to us, Goodings again reviews turbulent surface flows and their scaling dependence on the Froude Number and Reynolds Number. She shows the development of Froudian scaling criteria for the one-g model, such that

$$F_m = F_p$$

or

$$F_m = \frac{v_m^2}{g d_m} = \frac{q_m^2}{g d_m^3} = \frac{q_p^2}{g d_p^3} = F_p$$

From this Froudian relationship, the time scale is $\frac{t_m}{t_p} = 1/N^{1/2}$. However, Goodings concern is erosion due to seepage and thus, uses seepage discharge similitude of $\frac{q_m}{q_p} = 1/N$, which gives a time scale of $\frac{t_m}{t_p} = 1/N^{2/3}$. Only with a full scale model, i.e., $N = 1$, can the seepage similitude and surface flow (Froudian) similitude be maintained or where seepage flow is insignificant.

For a geometrically similar centrifuge model with reduced soil particle size such that $\left(\frac{D_{10m}}{D_{10p}} = 1/\sqrt{N}\right)$, then $\frac{q_m}{q_p} = 1/N$ (seepage discharge similitude). For model and prototype Froude Numbers to be equal, i.e.,

$$F_m = \frac{v_m^2}{N g d_m} = \frac{q_m^2}{N g d_m^3} = \frac{q_p^2}{g d_p^3} = F_p$$

From which $\frac{d_m}{d_p} = 1/N$, and with the Chezy equation as the descriptor of velocity yields $V_m = V_p$ and $\frac{t_m}{t_p} = 1/N$; which are identical for seepage flow. Goodings concludes that with a reduced particle size, a centrifuge model can maintain similitude for seepage flow and turbulent surface flow as long as the model Reynolds Number is in the same flow regime as the Reynolds Number of the full-scale prototype. However, Goodings also notes that maintaining Froudian similitude does not fully address sediment transport similitude.

In her 1984 article, Goodings also examined the initiation of erosion and sediment transport, showing that the initiation of erosion was dependent upon a critical shear stress, τ_c , required to dislodge a soil particle is

$$\tau_c = F_s g(\rho_s - \rho_w) D$$

where D a representative particle size, F_s is an "entrainment" function defined by $F_s = \rho_w dS/(\rho_s - \rho_w)D$, S is the slope of the energy line, d is the depth of uniform flow. The entrainment function can be related to a "particle" Reynolds Number, where particle diameter is the critical dimension in the Reynolds Number.

$$Re^* = \frac{\left(\frac{\tau_0}{\rho_w}\right)^{1/2} D}{\upsilon} = (gdS)^{1/2} D/\upsilon$$

The entrainment function is essentially constant for Re^* above about 500 because of the difficulty in "entraining" coarse particles. Goodings also recognized that the steep slope of a levee will also contribute to the instability of a surface particle and included a modified F_s accounting for the levee slope compared to the friction angle of the levee soil.

In a conventional one-g model, erosion will occur at a depth of flow

$$d > F_s\left(\rho_s - \frac{\rho_w}{\rho_w}\right) D/S = F_s(\rho_s - 1) D/S$$

Goodings states that if the model entrainment function is equal to the full scale entrainment function, $F_{sm} = F_{sp}$, and the depth of flow is scaled at *1/N*, then the all the soil particle diameters must also be scaled at *1/N*. Although explained in greater detail, this is similar to the conclusions reached in Goodings previous (1982) article.

In Goodings examination of high-g modeling, all of the criteria given for the one-g model in previous section must also be met for modeling in the high-g environment. She added the complexity of a cohesive soil, whose fine grains reduce the likelihood of developing a model with particle sizes modeled on *1/N* and therefore the prototype soil must be used in the model. Further, since no relationships had been developed for describing critical shear stress for cohesive soil, she argued that the shear stress must be modeled correctly. With a depth of flow in the centrifuge model equal to *1/N* of the full-scale, model and prototype velocities and shear stresses would be equal, satisfying model/prototype shear stress equality requirement.

Goodings also looks at sediment transport described by a combination of the entrainment function, particle size, and settling velocities. For correct transport modeling, Goodings argues that, for erosion initiation, the model and prototype entrainment functions must be equal, which requires that model/prototype soil particles be scaled at *1/N*. With the smaller particles, in a one-g model, the settling velocities will be $1/N^{1/2}$ less than the prototype, causing the rate of sediment discharge from the model to exceed the prototype by a factor of $N^{3/2}$. In other words, erosion in the model will proceed *N* times more quickly than the prototype because material is settling so slowly in the one-g model.

In her third article, Goodings (1985) reiterates the analysis and various scaling parameters provided in the 1982 and 1984 papers subsurface and surface flow. However, she adds a discussion of turbulent subsurface flow, groundwater surges, mass soil movement, and extensively discusses modeling of the phreatic surface. Of interest to us for surface flow erosion, Goodings notes that her experiments in a centrifuge model showed the entrainment function, as presented in her papers, overestimated the erosion and required calibration with model observations and then application to other high-g experiments. Goodings summarizes the 1985 paper, for our purposes, noting that similarity can only be maintained by reducing the soil particles of the model by a factor of *1/N*. However, this may conflict with the requirement to maintain critical soil characteristics, such as soil density, permeability, and capillary forces.

Dong, et al. (2001) provided a very concise review of the scaling laws for high-g modeling of soil transport by turbulent flow. They essentially start with the Buckingham-PI theory, listing the important parameters and then assembling them into dimensionless ratios or groupings. The dimensionless groupings include the Reynolds Number, Densimetric Froude Number, and ratios of soil density to water density and critical length to soil particle size. Using this approach to develop scaling criteria, Dong, et al. also reach Goodings' (1982, 1984, 1985) conclusion that the sediment size must be the same as the geometric scale. However, as discussed earlier, this criterion is very difficult to meet if the model is small. Using lighter weight material in the model can provide an adjustment to the densimetric Froude Number to help move toward the needed similitude relationship.

Dong, et al. (2001) argue that the forgoing analysis are largely applicable to river or stream bed stability and are only useful for weak flows and relatively stable bed forms. They also argue that local erosion is driven by "…strongly turbulent flows, with large amounts of sediment in suspension…" As a consequence, they suggest using particle fall velocities as a similitude criterion in a dimensionless parameter

$$\omega^* = \omega t/z$$

where ω is the average particle fall velocity, t and z are typical time and length scales of particle motion. Making this parameter equal in model and full scale prototype,

$$n_\omega n_t/n_z = 1$$

where n represents the ratio of model to prototype dimension and using particle scaling ratio, give a particle velocity scaling ratio similar to the conventional Froudian scaling criterion for velocity

$$n_\omega = \sqrt{n_g n_z}$$

Dong, et al. combines this relationship with particle fall velocity relationships presented by Hallermeier (1981), who develop a set of equations describing fall velocity for three size ranges of sand particles. This combination results in the following relationships for fall velocity similitude and grain size similitude:

Table 1. Fall Velocity-based particle scaling laws

Particle Size	Fall Velocity Scale n_ω	Grain Size Scale n_D
D<0.14 mm	$\frac{n_\Delta n_g {n_D}^2}{n_\nu}$	$\frac{{n_L}^{1/4}}{{n_g}^{1/4}}$
0.14 mm<D<0.9 mm	$\frac{{n_\Delta}^{0.7} {n_g}^{0.7} {n_D}^{1.1}}{{n_\nu}^{0.4}}$	$\frac{{n_L}^{0.45}}{{n_g}^{0.18}}$

They point out that these laws are not precise because Hallermeier's settling velocities were based on clear water, not highly sediment-laden turbulent flow.

Using the same fluid and particle type in the model as in the prototype, i. e., $n_v = n_\Delta = 1$ and $z = L$, and making these substitutions into $n_\omega = \sqrt{n_g n_z}$, particle scaling can easily be derived as shown in the table in the previous paragraph. For a standard centrifuge model where $n_g = 1/n_L = n$, and the smaller grain size, the sediment scale becomes $n_\mathrm{D} = 1$ and $n_\mathrm{D} = n^{0.63}$ for the larger particle size. These similitude laws provide a much more feasible basis for centrifuge modeling of surface erosion compared to the previous development of particle size scaling.

Garnier, et al. (2007) provided a compendium of scaling laws for centrifuge modeling. For surface erosion, their summary cites the efforts of Goodings discussed earlier, identifying the conflict be sediment size scaling for different flow regimes. No other efforts are included or evaluated, including the analysis by Dong, et al. (2001).

Gross, et al. (2010) conduct one-g physical modeling of levee erosion for validation of numerical simulations. They specifically examined the development of rills and gullies to determine the response of the levee embankment to overtopping, breaching, and ultimate failure. They tested different soils (sand and sand/clay mixture) in different embankment cross sections, including a half-levee and full-levee. Data were collected with a multi-beam 3-D laser scanner before and after each experiment, showing the progression of the surface erosion. Of specific interest were the dimensions of the rills and erosion patterns and the time to rill formation and breaching under a range of flow rates with different soil mixtures. Ultimately, Gross, et al. intended to take this type of experimentation to high-g centrifuge modeling.

Bezuijen and Steedman (2010) performed a literature study on the centrifuge scaling of hydraulic process. Considering that the flow in open channel will most probably be turbulent, one of the expressions quoted for soil erosion was proposed by Van Rhee (2002, After Bezuijen and Steedman, 2012). For 1-g conditions, the erosion rate is:

$$E = k * \rho_s \sqrt{\Delta g D}\,\left[\frac{f_0}{8\Delta g D}\,(u^2 - u_{cr}^2)\right]^\alpha$$

where E is the erosion rate, k a coefficient, ρ_s the density of the grain is $\Delta = (\rho_s - \rho)/\rho$ with ρ the liquid density, D the average diameter the grain, f_0 the Darcy-Weisbach friction coefficient, u the flow velocity, u_{cr} the critical velocity, and α, a constant (1.5). Assuming that the assumption was correct, Bezuijen and Steedman (2010) suggested using the expression by scaling down the grain size by N. This idea may work for cohesionless material but may not work for cohesive material. They further suggested exploring the dimensionless shear stress, Shields parameter θ, which is the dimensionless critical shear stress. For centrifuge N-g test as the following expression,

$$\theta = \frac{\tau}{(\rho_s - \rho)\,NgD} = \frac{u_*^2}{\Delta NgD}\; with\, u_* = \sqrt{\frac{f}{8}\,u}$$

Van Beek et al. (2010) studied the effects of scaling in centrifuge test for backward erosion piping problems. They hypothesize that there is no effect on the critical head, however the test results show that the critical head decrease as g-level increase, in this case change the specimen length. They discussed that it was caused by transitional flow in channel. Marot et al. (2012) considered to model the hydraulic loading by the fluid expended energy, and they found that critical head was not affected by the specimen length. While not directly related to the embankment breach process, these finding could be used when studying breaching by backward erosion piping (Marot et al. 2012).

Silva-Araya, et al. (2010) also experimentally studied levee erosion and breaching in a conventional one-g model with two different soils: sandy loam and clay loam. On these experiments, overtopping was the cause of failure, starting with the initiation of head cuts along the slope of the levee. Once one of the head cuts progressed to the upstream side of the levee, the flow along this path concentrated and deepened the gully with the sidewalls ultimately collapsing and caving into the flow. They conclude that even a small amount of clay (15 percent) can enhance the cohesive behavior of the soil and can slow erosion due to overtopping.

Kamalzare, et al. (2012) actually conducted one-g and high-g physical model studies of levee erosion. They studied erosion patterns for a single mixture of soil (25 percent clay and 75 percent sand). Their primary focus for these experiments was to provide validation of numerical simulations of rills and gullies in the levee the time required for rill initiation and progression to the levee crest. They offer no scaling criteria, but simply state that erosion will occur much faster for high-g modeling than for one-g models.

Their experiments were conducted under an acceleration of 20 g (likely 1:20 scale model for conventional centrifuge modeling) with experimental documentation by high resolution high video camera for comparison with digital simulations. Their modeled levee was equivalent to 17.5 m long, 178 m high, and 7.90 m wide. For one experiment, they show a flow rate of 0.56 l/m (presumably model flow) with breaching occurring in about 5 minutes, equivalent to 100 min full scale (time scale of 1:20, not $1{:}(20)^{1/2}$, typical for surface flow modeling). They also compare their observed breaches with a FEMA levee breach equation (but offer no reference).

Based on their centrifuge experiments, they find additional high-g experiments are needed. They also recommend even higher resolution cameras for documenting the erosion because of the rapidity of erosion under high-g modeling. They experienced erosion occurring along the edges of their model, presumably also the end-walls of the model box and suggested constructing the model with a slight curve to avoid breaching at the edges of the model.

5 SUMMARY AND RECOMMENDATIONS

Goodings' (1982, 1984, 1985) set of articles did not provide much optimism regarding the potential for erosion experiments under high-g conditions, because of the similitude requirement that particle sizes in the model be scaled according to the length scale of the model. This would require an extremely large model to avoid significant changes in the soil characteristics. However, her focus was also on subsurface water movement in addition to surface erosion. Garnier, et al.'s (2007) "Catalogue of Scaling Laws…" offered no extension of this analysis or conclusion.

Dong, et al. (2001), however, looks at a different approach for scaling the local erosion occurring as a consequence of levee overtopping and arrives at different scaling criteria for particles sizes, which seems to be much more feasible for high-g modeling. Kamalzare, et al. (2012) appear to be unfettered by the need to examine scaling criteria, and jump head-long into one-g and high-g modeling of levee breaching. Scaling model observations, such as time to initiation of surface erosion (rill and small gullies), time to full breach, and volumetric rate of erosion, may be problematic without appropriate scaling criteria. Even so, high-g comparative modeling of surface erosion could prove invaluable, since there is so little experience in high-g erosion modeling.

Based on this abbreviated review of past efforts at modeling surface erosion in centrifuge models, we recommend that an experimental program be designed to examine different aspects of levee breaching under high-g conditions. The program should be aimed, initially, at comparative modeling to evaluate the effects of different scaling laws that are based on different assumptions about the erosion process, i.e., Goodings' scaling versus Dong, et al.'s scaling. The program should be phased in such a way to allow redirection if the results prove fruitless.

6 REFERENCES

Bezujien, A., Steedman, R.S. (2010). Scaling of hydraulic process. Phisical Modelling in Geotechnics-Springman, Laue& Seward (eds) 2010 Taylor & Francis Group, London.

Dong, P., Newson, T. A., Davies, M. C. R., and Davies, P. A. (2001). "Scalling laws for Centrifuge Modeling of Soil Transport by Turbulent Fluid Flows," *International Journal of Physical Modelling in Geotechnics*, vol 1, pp 41-45.

Garnier, J., Gaudin, C., Springman, S. M., Culligan, P. J., Goodings, D., Konig, D., Kutter, B., Phillips, R Randolph, M. F., and Thorel, L. (2007). "Catalogue of Scaling Laws and Similitude Questions in Geotechnical Centrifuge Modeling," *International Journal of Physical Modeling in Geotechnics*, Issue 3, pp 1-23.

Goodings, D. J., (1982). "Relationships for centrifugal modeling of seepage and surface flow effects on, embankment dams," *Geotechnique*, vol 32, pp 149-152

Goodings, D. J., (1984). "Geotechnical Centrifuge Modeling of Soil Erosion," *Transportation Research Record* 998, pp 1-6

Goodings, D. J., (1985). "Relationships for modeling water effects in geotechnical centrifuge models," *Proceedings of a Symposium on the Application of Centrifuge Modeling to Geotechnical Design.* pp. 1-24

Gross, J. A., Stuetzle, C. S., Zhongxiam, C., Cutler, B., Franklin, W. R., Zimmie, T. F. (2010) "Simulating levee Erosion with Physical Modeling Validation," *ASCE Proceedings of International Conference on Scour and Erosion (ICSE-5)*, November 7-10, San Francisco, CA, pp 326-337.

Hallermeier, R. J. (1981). "Terminal settling velocity of commonly occurring sand grains," Sedimentology, vol 28, No. 6, pp 859-865

Hunt, S. L., Hanson,G. J., Cook, K. R., and Kadavy, K. C. (2005). "Breaching widening observation from earthen embankment tests," *Transaction of the American Society of Agricultural Engineers*, Vol 48(3), pp 1115-1120

Kamalzare, M., Stuetzle, C., Zhongxian, C., Zimmie, T. F., Cutler, B., and Franklin, W. R. (2012). "Validation of Erosion Modeling: Physical and Numerical," *ASCE GeoCongress* 2012, pp 710-719.

Marot, D., Le V.D., Garnier, J., Thorel, L., Audrain, P. (2012). Study of scale effect in an internal erosion mechanism. European Journal of Environmental and Civil Engineering, Vol. 16, No.1, pp.1-19.

Silva-Araya, W. F., Alva-Solari, L. H., and Chaudhry, M. H. (2010). Experimental Study of Levee Breach," Proceedings of 21st Century Watershed Technology: Improving Water Quality and Environment, 21-14 February 2010, Costa Rica, ASABE publication number 701P0210cd.

Van Beek, V. M., Bezuijen, A. Zwanenburg, C. (2010). Piping: Centrifuge experiments on scaling effects and levee stability. Physical Modelling in Geotechnics-Springman, Laue & Seward (eds) 2010 Taylor & Francis Group, London.